# Three-Photon Decay of Positronium $^1S$ State as a Test of Charge Conjugation Invariance

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In order to make comparisons with a possible experiment to test charge conjugation invariance, the angular distribution of decay photons is estimated for the forbidden $3\gamma$ decay of the $^1S$ state of positronium using the simplest matrix element satisfying Lorentz, gauge, and *TCP* invariance.

## 1. INTRODUCTION

ALTHOUGH the charge conjugation invariance of the electromagnetic interactions is generally well accepted, there does not seem to be unambiguous *direct* experimental evidence available to support this fact. The observed equality of the electron and positron masses is predicted by $C$ invariance but, on the other hand, is also predicted by $TCP$ or $CP$ invariance even if $C$ is not conserved.[1] Similarly, the fact that the magnetic moment of a particle is equal and opposite to the magnetic moment of its antiparticle is predicted by $TCP$ or $CP$ invariance[2] alone. It is well known, on the other hand, that $C$ invariance imposes restrictions on the decay of positronium into photons. Essentially[3] this is because both positronium and the multiphoton system are neutral so that positronium states are eigenfunctions of the operator $[U_c,Q]$ with eigenvalue zero, i.e.,

$$[U_c,Q]|\Psi_{\text{posit.}}\rangle=0\,.$$

The charge conjugation parity of $n$ photons is $(-1)^n$ while the charge conjugation parity of positronium is $(-1)^{l+s}$, where $l$ is the orbital and $s$ the spin angular momentum. The states most readily studied are the $^1S_0$ (para) state and the $^3S_1$ (ortho) state. The conservation of charge parity implies that the ortho state can only decay into an odd number of photons. However, since it is impossible for 2 photons in their center-of-momentum system to have total angular momentum one, the prohibition of $^3S_1\rightarrow 2\gamma$ is provided by angular-momentum conservation. Thus, the decay of ortho positronium does not furnish a test of $C$ invariance. Parapositronium is required by $C$ invariance to decay into an even number of photons; specifically, the $^1S_0\rightarrow 3\gamma$ decay is not allowed. No experiment has been performed to look for this mode although experiments[4] have been made to investigate the quenching of the ortho state, observing the cessation of delayed photons (corresponding to the much slower ortho decay) upon the addition of an ortho-para converting gas such as NO. In this way the delayed photons can be practically eliminated, indicating that either the nonallowed decay is very fast or else that it is very small. To obtain a more quantitative result it would be desirable to look at the angular distribution for three photons. By comparing the experimental results with the allowed $^3S_1\rightarrow 3\gamma$ angular distribution,[5] an upper limit can be set on the amount of allowed $C$ noninvariance. In this connection it is interesting to estimate the angular distribution which would result if the $^1S_0\rightarrow 3\gamma$ process could take place. To calculate this an interaction Lagrangian which violates $C$ invariance must be assumed.

## 2. CHOICE OF MATRIX ELEMENT

We look for the simplest Lorentz and gauge-invariant "equivalent" local Lagrangian which will give $3\gamma$ decay in first order. This will be of the form

$$L_I(x)=\bar{\psi}O\psi(FFF)\,, \tag{1}$$

where $O=1$, $\gamma_5$, $\gamma_\mu$, $\gamma_5\gamma_\mu$, $\sigma_{\mu\nu}$ and $(FFF)$ is a combination of three electromagnetic four-tensors, possibly involving derivatives.

Since $(FFF)$ is odd under charge conjugation, $O$ must be chosen so that $\bar{\psi}O\psi$ is even. There are three possibilities[6]:

$$O=1,\ \gamma_5,\ \gamma_5\gamma_\mu\,.$$

The choice can be further narrowed by noting that the matrix elements corresponding to several Lagrangians vanish because of the antisymmetry of the initial positronium state. If the initial state is expanded in the center-of-mass system (see, for example, Jauch and Rohrlich[7]) as

$$|\Psi_i\rangle=\sum_{r,s}\int d^3p\,\phi(\mathbf{p},rs)a_r^\dagger(\mathbf{p})b_s^\dagger(-\mathbf{p})|0\rangle\,, \tag{2}$$

where $r$=electron spin index, $s$=positron spin index, and

$$\phi(\mathbf{p},11)=\phi(\mathbf{p},22)=0\,,$$

$$\phi(\mathbf{p},12)=-\phi(\mathbf{p},21)\,,$$

[1] G. Luders and B. Zumino, Phys. Rev. **106**, 385 (1957); R. E. Marshak and E. C. G. Sudarshan, *Introduction to Elementary Particle Physics* (Interscience Publishers, Inc., New York, 1961) p. 109.

[2] K. Gotow and S. Okubo, Phys. Rev. **128**, 1921 (1962).

[3] See, for example, R. E. Marshak and E. C. G. Sudarshan, Ref. 1, pp. 94–97.

[4] Martin Deutsch, Phys. Rev. **82**, 455 (1951). See also Martin Deutsch, in *Progress in Nuclear Physics*, edited by D. R. Frisch (Pergamon Press, Inc., New York, 1953).

[5] This is essentially given, for example, in J. M. Jauch and F. Rohrlich, *The Theory of Photons and Electrons* (Addison-Wesley Publishing Co., Inc., Reading, Massachusetts, 1955), p. 273.

[6] See R. E. Marshak and E. C. G. Sudarshan, Ref. 1, p. 90.

[7] J. M. Jauch and F. Rohrlich, Ref. 5, pp. 283–289.

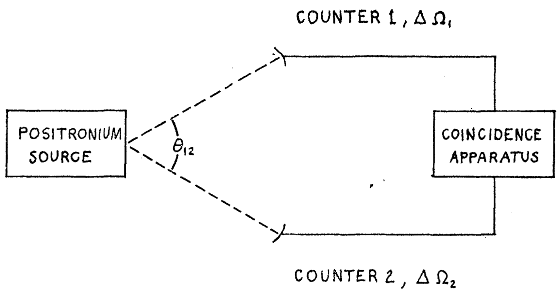


FIG. 1. Schematic diagram of experiment.

for the $^1S$ state, and if the leading term of the expansion of the transition matrix element in powers of $\mathbf{p}/m$ is retained,

$$M=\sum_{r,s} M_{rs}(\mathbf{p}=-\mathbf{p}=0)\int \phi(\mathbf{p},rs)d^3p\,; \qquad (3)$$

then in our case:

$$M\propto \mathrm{Tr}[\gamma_5 L_{\mathrm{I}}(\mathbf{p},\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)(1+\gamma_4)]\,, \qquad (4)$$

where $\mathbf{k}_1$, $\mathbf{k}_2$, $\mathbf{k}_3$ = momenta of decay photons.

Since $\mathrm{Tr}(\gamma_5\gamma_4)=0$, $M$ vanishes for $O=1$. However, $M$ is nonvanishing for $O=\gamma_5$, while for $O=\gamma_5\gamma_\mu$ $M$ contains a factor $\delta_{\mu4}$.

Now we examine the various forms of $(FFF)$ in order of increasing complexity. The simplest case is where $(FFF)$ involves no derivatives. There are three possibilities, all of which are identically zero:

0(i). In $F_{\mu\nu}F_{\mu\lambda}F_{\nu\lambda}$ denote $G_{\nu\lambda}=F_{\mu\nu}F_{\mu\lambda}$ and note that $G_{\nu\lambda}$ is symmetric on $\nu\leftrightarrow\lambda$ interchange while $F_{\nu\lambda}$ is antisymmetric;

0(ii). In $\epsilon_{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}F_{\mu\mu}$, $F_{\mu\mu}=0$;

0(iii). In $\epsilon_{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\mu}F_{\delta\mu}$ denote $H_{\gamma\delta}=F_{\gamma\mu}F_{\delta\mu}$ and use the argument of (i).

We next try forms of $(FFF)$ containing one derivative. For Lorentz invariance these must be coupled to $\bar\psi\gamma_5\gamma_\mu\psi$. If $(FFF)$ contains the $\epsilon_{\alpha\beta\gamma\delta}$ symbol, it will be a pseudovector and hence $L_{\mathrm{I}}$ will be a true scalar. Then by the $TCP$ theorem $L_{\mathrm{I}}$ will not be invariant under time inversion. On the other hand, if $(FFF)$ does not contain $\epsilon_{\alpha\beta\gamma\delta}$ it will be a true vector, $L_{\mathrm{I}}$ will be a pseudoscalar, and will be invariant under time inversion. Since violation of time-reversal invariance has apparently never been observed, we look for a suitable $(FFF)$ not containing $\epsilon_{\alpha\beta\gamma\delta}$. [Similar reasoning incidentally would also eliminate cases 0(ii) and 0(iii) of the previous form.] There are now the possibilities:

I(i). $(\partial_\mu F_{\alpha\beta})F_{\alpha\gamma}F_{\beta\gamma}=0$, since $G_{\alpha\beta}=F_{\alpha\gamma}F_{\beta\gamma}$ is symmetric on $\alpha\leftrightarrow\beta$ interchange;

I(ii). $$F_{\beta\mu}F_{\alpha\gamma}(\partial_\alpha F_{\beta\gamma})=-(\partial_\alpha F_{\beta\mu})F_{\alpha\gamma}F_{\beta\gamma}-F_{\beta\mu}(\partial_\alpha F_{\alpha\gamma})F_{\beta\gamma}+\partial_\alpha(F_{\beta\mu}F_{\alpha\gamma}F_{\beta\gamma})\,.$$

The second term on the right-hand side is zero while the third "surface" term gives no contribution. Thus (2) is essentially equivalent to the following;

I(iii). $(\partial_\alpha F_{\beta\mu})F_{\alpha\gamma}F_{\beta\gamma}$ is, therefore, a unique choice if we do not consider more complicated forms of $(FFF)$.

Alternately, we may obtain the same result by recognizing that the $^1S$ state of positronium can be described by a single pseudoscalar "field" $B(x)$. Then Eq. (1) would be replaced by $L_{\mathrm{I}}\propto[B(x)](FFF)$ where $[B(x)]$ may involve derivatives. Similar reasoning would show that the simplest possible nonvanishing form is $(\partial_\mu B)(\partial_\alpha F_{\beta\mu})F_{\alpha\gamma}F_{\beta\gamma}$. In momentum space in the center-of-mass system $\partial_\mu B(x)=2m\delta_{\mu4}$. The matrix element obtained from this is, therefore, also suitable for a discussion of particle conjugation invariance and the forbidden decay,[8] $\pi^0\to 3\gamma$.

Thus, we will calculate with the interaction Lagrangian:

$$L_{\mathrm{I}}(x)=\mathrm{const}\ \bar\psi\gamma_5\gamma_\mu\psi(\partial_\alpha F_{\beta\mu})F_{\alpha\gamma}F_{\beta\gamma}\,. \qquad (5)$$

Since the orbital momenta of the initial electron and positron are small compared to the outgoing photon momenta we take as the equations of energy-momentum conservation

$$\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3=0\,,\qquad \omega_1+\omega_2+\omega_3=2m\,. \qquad (6)$$

The transition matrix element Eq. (4), using the $L_{\mathrm{I}}$ of Eq. (5), is essentially the fourth component of I(iii) written out in momentum space and symmetrized with respect to all three photons:

$$\begin{aligned}M=\mathrm{const}&\sum_{\substack{\text{permutations}\\ \text{of }1,2,3}} k_\alpha^{(1)}(k_4^{(1)}A_\beta^{(1)}-k_\beta^{(1)}A_4^{(1)})(k_\gamma^{(2)}A_\alpha^{(2)}-k_\alpha^{(2)}A_\gamma^{(2)})(k_\gamma^{(3)}A_\beta^{(3)}-k_\beta^{(3)}A_\gamma^{(3)})\\
=\mathrm{const}\{&(\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\varepsilon}_2)[(m-\omega_2)(\mathbf{k}_1\cdot\boldsymbol{\varepsilon}_3)(\mathbf{k}_2\cdot\mathbf{k}_3)+(m-\omega_1)(\mathbf{k}_1\cdot\mathbf{k}_3)(\mathbf{k}_2\cdot\boldsymbol{\varepsilon}_3)]\\
&+(\boldsymbol{\varepsilon}_2\cdot\boldsymbol{\varepsilon}_3)[(m-\omega_2)(\mathbf{k}_3\cdot\boldsymbol{\varepsilon}_1)(\mathbf{k}_2\cdot\mathbf{k}_1)+(m-\omega_3)(\mathbf{k}_3\cdot\mathbf{k}_1)(\mathbf{k}_2\cdot\boldsymbol{\varepsilon}_1)]\\
&+(\boldsymbol{\varepsilon}_1\cdot\boldsymbol{\varepsilon}_3)[(m-\omega_1)(\mathbf{k}_1\cdot\mathbf{k}_2)(\mathbf{k}_3\cdot\boldsymbol{\varepsilon}_2)+(m-\omega_3)(\mathbf{k}_1\cdot\boldsymbol{\varepsilon}_2)(\mathbf{k}_3\cdot\mathbf{k}_2)]\\
&-m[(\boldsymbol{\varepsilon}_1\cdot\mathbf{k}_2)(\boldsymbol{\varepsilon}_2\cdot\mathbf{k}_3)(\boldsymbol{\varepsilon}_3\cdot\mathbf{k}_1)+(\boldsymbol{\varepsilon}_1\cdot\mathbf{k}_3)(\boldsymbol{\varepsilon}_2\cdot\mathbf{k}_1)(\boldsymbol{\varepsilon}_3\cdot\mathbf{k}_2)]\}\,, \qquad (7)\end{aligned}$$

where $\boldsymbol{\varepsilon}_1$, $\boldsymbol{\varepsilon}_2$, $\boldsymbol{\varepsilon}_3$ = photon polarization unit vectors. In obtaining the above expression we have made use of Eqs. (6).

[8] Inconclusive results were obtained in the experiment of Ely and Frisch [R. P. Ely and D. H. Frisch, Phys. Rev. Letters **3**, 565 (1959)].

### 3. ANGULAR DISTRIBUTION OF DECAY PHOTONS

We are interested in an experiment of the type shown in Fig. 1. The quantity of interest is, therefore,

$$\text{coincidence rate}=\alpha f(\theta_{12})\Delta\Omega_1\Delta\Omega_2\,, \tag{8}$$

where $\Delta\Omega_{1,2}$= solid angle subtended by counter 1, 2; $\alpha$= normalization constant; and $f(\theta_{12})$ is given by

$$f(\theta_{12})=\sum_{\text{photon polarizations}}\int_0^m \frac{|M|^2}{\omega_1\omega_2\omega_3}\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\times\delta(\omega_1+\omega_2+\omega_3-2m)\omega_1^2d\omega_1\omega_2^2d\omega_2d^3k_3$$

$$=\sum\int_0^m |M|^2\left(\frac{\omega_1\omega_2}{\omega_3}\right)\left|\frac{\partial\omega_2}{\partial(\omega_1+\omega_2+\omega_3)}\right|d\omega_1$$

$$=\sum\int_0^m \frac{|M|^2\omega_1\omega_2}{2m-\omega_1(1-\cos\theta_{12})}, \tag{9}$$

$M$ being given by Eq. (7) apart from the constant factor which is absorbed into $\alpha$ of Eq. (8).

We shall advance the evaluation of Eq. (9) to the point where only a straightforward but involved integral need be calculated and shall obtain an approximate valid for small $\theta_{12}$. The diagrams of Fig. 2 describe the kinematics of the decay photons and illustrate the choice of 2 linearly independent polarization vectors for each photon. Evidently

$$\boldsymbol{\varepsilon}_i^{(\alpha)}\cdot\boldsymbol{\varepsilon}_j^{(\beta)}=0\,,\quad \boldsymbol{\varepsilon}_i^{(\beta)}\cdot\boldsymbol{\varepsilon}_j^{(\beta)}=1\,,\quad \boldsymbol{\varepsilon}_i^{(\beta)}\cdot\mathbf{k}_j=0\,,$$
$$\boldsymbol{\varepsilon}_i^{(\alpha)}\cdot\mathbf{k}_j=\pm\omega_j\sin\theta_{ij}\,,\quad \mathbf{k}_i\cdot\mathbf{k}_j=\omega_i\omega_j\cos\theta_{ij}\,. \tag{10}$$

$M$ is nonvanishing for only three of the eight possible choices of polarization directions. These are, using

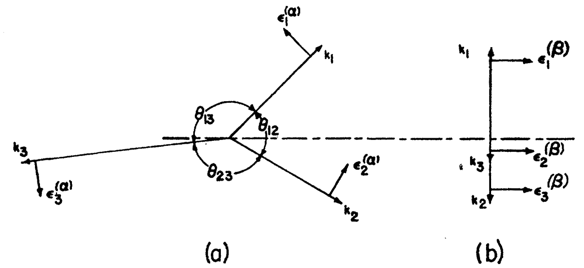


FIG. 2. (a) Top view of decay plane; (b) edge view of decay plane.

Eqs. (10),

(a). $(\boldsymbol{\varepsilon}_1^{(\alpha)},\boldsymbol{\varepsilon}_2^{(\beta)},\boldsymbol{\varepsilon}_3^{(\beta)})$:

$$M_a=\omega_1\omega_2\omega_3[(m-\omega_2)\sin\theta_{13}\cos\theta_{12}-(m-\omega_3)\cos\theta_{13}\sin\theta_{12}];$$

(b). $(\boldsymbol{\varepsilon}_1^{(\beta)},\boldsymbol{\varepsilon}_2^{(\alpha)},\boldsymbol{\varepsilon}_3^{(\beta)})$:

$$M_b=\omega_1\omega_2\omega_3[(m-\omega_3)\sin\theta_{12}\cos\theta_{23}-(m-\omega_1)\cos\theta_{12}\sin\theta_{23}];$$

(c). $(\boldsymbol{\varepsilon}_1^{(\beta)},\boldsymbol{\varepsilon}_2^{(\beta)},\boldsymbol{\varepsilon}_3^{(\alpha)})$:

$$M_c=\omega_1\omega_2\omega_3[(m-\omega_1)\cos\theta_{13}\sin\theta_{23}-(m-\omega_2)\sin\theta_{13}\cos\theta_{23}]\,. \tag{11}$$

Equation (11) can be put in more useful form with the aid of the following kinematic identities, obtained by crossing and dotting $\mathbf{k}_1$ and $\mathbf{k}_2$ into the first of Eqs. (6):

$$\sin\theta_{13}=(\omega_2/\omega_3)\sin\theta_{12}\,,\quad \cos\theta_{13}=-(\omega_1+\omega_2\cos\theta_{12})/\omega_3\,,$$
$$\sin\theta_{23}=(\omega_1/\omega_3)\sin\theta_{12}\,,\quad \cos\theta_{23}=-(\omega_2+\omega_1\cos\theta_{12})/\omega_3\,. \tag{12}$$

Using Eqs. (12) in Eqs. (11) and putting the result in Eq. (9) gives for the angular distribution function

$$f(\theta_{12})=\sin^2\theta_{12}\int_0^m\frac{\omega_1^3\omega_2^3}{\omega_3^2}\frac{1}{2m-\omega_1(1-\cos\theta_{12})}\{[\omega_2(m-\omega_2)(\omega_2+\omega_1\cos\theta_{12})-\omega_1(m-\omega_1)(\omega_1+\omega_2\cos\theta_{12})]^2$$
$$+\omega_3^2[\omega_2(m-\omega_2)\cos\theta_{12}+(m-\omega_3)(\omega_1+\omega_2\cos\theta_{12})]^2+\omega_3^2[\omega_1(m-\omega_1)\cos\theta_{12}+(m-\omega_3)(\omega_2+\omega_1\cos\theta_{12})]^2\}d\omega_1\,,$$

where

$$\omega_2=2m(m-\omega_1)/[2m-\omega_1(1-\cos\theta_{12})]\,,$$
$$\omega_3=2m-\omega_1-\omega_2\,. \tag{13}$$

Equation (13) is the final result which may be integrated if desired. The first term in the curly brackets corresponds to both polarization vectors perpendicular to the decay plane ($M_c$). The second and third terms correspond to one polarization vector in and the other perpendicular to the decay plane ($M_a$ and $M_b$).

We now estimate $f(\theta_{12})$ for small $\theta_{12}$. The essential approximation is

$$\omega_3\approx m\,. \tag{14}$$

This is valid to within about 2% up to $\theta_{12}=30°$.

Approximating the part of Eq. (13) corresponding to $M_b$ and $M_c$ gives easily:

$$f_{\parallel,\perp}(\theta_{12})\approx 2\cos^2\theta_{12}\sin^2\theta_{12}\int_0^m\frac{8m^3\omega_1^5(m-\omega_1)^5}{(2m-A\omega_1)^4}d\omega_1\,,$$

where

$$A=1-\cos\theta_{12}\,. \tag{15}$$

Approximating the part of Eq. (13) corresponding to $M_a$ gives

$$f_{\perp,\perp}(\theta_{12})\approx\sin^2\theta_{12}(1-\cos\theta_{12})^2\times\int_0^m\frac{8m^3\omega_1^5(m-\omega_1)^5}{(2m-A\omega_1)^4}\left(\frac{4\omega_1}{m}\right)d\omega_1\,. \tag{16}$$

We note that the integrands in Eq. (15) and Eq. (16) differ only by a factor of $4\omega_1/m$. We approximate this by 2 over the entire range of integration. To take some amount of the angular dependence introduced by the denominator of the integrand of Eqs. (15) and (16) we use a modulation factor of $1/(1-\frac{1}{4}A)^4$. Then the desired small-angle estimate is

$$f(\theta_{12})\approx \text{const}\frac{\sin^2\theta_{12}(1-2\cos\theta_{12}+2\cos^2\theta_{12})}{(1-\frac{1}{4}A)^4}. \quad (17)$$

### 4. EXPERIMENTAL CONSIDERATIONS

Many details are discussed in the review article by Deutsch.[4] The contribution of the $^3S_1\rightarrow 3\gamma$ decay to the angular distribution can be greatly suppressed by using NO for quenching. Furthermore, this contribution[5] has the characteristic feature that it takes on its maximum value when two photons come out nearly together, i.e., for $\theta_{12}=0°$. On the other hand, the distribution given by Eq. (17) is zero for $\theta_{12}=0°$. Therefore, the known effect of the allowed $3\gamma$ decay can be unambiguously subtracted away to obtain an estimate of the strength of any possible interaction given by Eq. (5).

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# Elastic Scattering of Pseudoscalar Mesons and $SU_n$ Symmetry*

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We derive the crossing matrix for scattering $X_n+X_n\rightarrow X_n+X_n$ of a multiplet of scalar mesons $X_n$ which transform according to the regular representation of $SU_n$, for all $n$. For $n=3$, or octet symmetry, in the limit of neglecting inelastic coupled channels we find that, if the $J^P=1^-$ $8'$ channel resonates, so also should the other two $1^-$ channels 10 and $\overline{10}$. These decuplets resonate at essentially the same mass as the $8'$, barring what are probably small corrections from crossed $0^+$ channels 8 and 27. Similarly, for general $n$, all $J^P=1^-$ channels should resonate together. Application of the Gell-Mann and Okubo mass-splitting formula to the degenerate $8'$, 10, and $\overline{10}$ leaves the 10 and $\overline{10}$ in energy regions which have been explored in experiments, at least if one assumes the 725-MeV $(K\pi)$ resonance is a member of the decuplet. However, effects from coupled inelastic channels such as $X_n+V_n\rightarrow X_n+V_n$ ($V_n$ is $J^P=1^-$ multiplet) may remove the mass degeneracy in the limit before symmetry-breaking effects are introduced. The crossing relations for $n=3$ and $V_3=8'$, 10, $\overline{10}$ are examined and shown to be consistent with this explanation. For $n=3$, the $X_3+X_3\rightarrow X_3+X_3$ crossing relations favor a pseudoscalar, rather than a scalar octet $X_3$.

## I. INTRODUCTION

RECENTLY, two principles have been used with rather striking success to classify the multitude of newly discovered baryonic resonances.[1] These are (a) that the resonances fall into sets which form irreducible representations of $SU_3$, the group of $3\times 3$ unimodular matrices[2]; (b) that the operator which gives the symmetry-breaking mass-splittings between the members of each set is proportional to the eighth component of unitary spin.[3] The latter principle will receive strong confirmation if two baryon resonances which it predicts, the $S=-2$, $\Xi_\gamma$ at 1600 MeV and the $S=-3$, $\Omega_\delta^-$ at 1676 MeV, are discovered.

In the present paper, these two ideas are joined to a third; namely, that strongly interacting particles, in virtue of crossing symmetry, are self-generating (the "bootstrap" philosophy of Chew and Frautschi[4]). These three are then applied to the low-energy vector mesons $V=(\rho,K^*,\bar{K}^*,\omega)$ considered as resonances in the scattering $XX\rightarrow XX$ of pseudoscalar mesons $X=(\pi,K,\bar{K},\eta)$. What is here done for $SU_3$ is the equivalent of the problem for $SU_2$, isotopic spin, where $X=(\pi^+,\pi^0,\pi^-)$ and $V=(\rho_+,\rho_0,\rho_-)$; and the $V$ resonance is self-generating, since the force which produces $V$ is the exchange of $V$.[5] The $SU_3$ case differs from the $SU_2$ in that the strength of the symmetry-breaking interactions make an answer derived in the limit of exact $SU_3$ symmetry

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